$$V(x) = -\frac{3}{2}p + \frac{29}{8} - \left(-\frac{p}{2} + \frac{13}{8}\right)^2$$

$$= -\frac{3}{2}p + \frac{29}{8} - \left(\frac{p^2}{4} - \frac{26}{16}p + \frac{13^2}{8^2}\right)$$

$$= -\frac{3}{2}p + \frac{29}{8} - \frac{p^2}{4} + \frac{26}{16}p - \frac{13^2}{8^2}$$

$$= -\frac{p^2}{4} + \frac{p}{8} + \frac{63}{64}$$

a variância é mínima para $p=0 \vee p = 1/2$

2.11) $X: \begin{cases} 0 & 2500 & 25000 \\ \frac{9989}{10000} & \frac{1}{1000} & \frac{1}{10000} \end{cases}$

a)

b) $P(X=0) = 0,9989$

c) $P(X=2500) + P(X=25000) = 0,0011$

d) $E(X) = 5$ u.m

$V(X) = 68750 - 5^2 = 68725$ u.m^2

$E(X^2) = 68750$

$CV(X) = \frac{\sqrt{68725}}{5} \times 100 = 5243,09\%$

2.13) a) $f(x) = \begin{cases} 0 & , x < 0 \\ x e^{-x} & , x \geq 0 \end{cases}$

$$(1-(x+1)e^{-x})' = -e^{-x} - (x+1)\cdot -e^{-x}$$

$$= e^{-x}(-1+x+1)$$

$$= e^{-x}.x$$

$\int fg = Fg - \int Fg'$

b) $$E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^{+\infty} \underset{g}{x^2}\,\underset{f}{e^{-x}}\,dx = \overbrace{\left[-e^{-x}x^2\right]_0^{+\infty}}^{\to 0} - \int_0^{+\infty} -\underset{F}{e^{-x}}\cdot \underset{g}{2x}\,dx$$

$$= \overbrace{\left[-e^{-x}\cdot 2x\right]_0^{+\infty}}^{\to 0} - \int_0^{+\infty} -e^{-x}.2\,dx = -2\left[e^{-x}\right]_0^{+\infty} = -2\left(0 - \frac{1}{e^0}\right) = 2$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x)\,dx = \int_0^{+\infty} x^3 e^{-x}\,dx = 6$$

$$V(X) = 6 - 2^2 = 2$$

2.15) $f(x) = \begin{cases} k\sin(x) & , 0 \leq x \leq \pi \\ 0 & , \text{c.c.} \end{cases}$

a) 1. $f(x) \geq 0, \forall_{x \in \mathbb{R}}$

para $\boxed{0 \leq x \leq \pi}$

$k\sin(x) \geq 0 \Rightarrow k \geq 0$

2. $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \Leftrightarrow \int_0^{\pi} k\sin(x)\,dx = 1 \Leftrightarrow k\left[-\cos(x)\right]_0^{\pi} = 1$$

$$\Leftrightarrow k(1+1) = 1 \Leftrightarrow 2k = 1 \Rightarrow k = \frac{1}{2}$$

b) $$E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^{\pi} \frac{x}{2} \underbrace{\sin(x)}_{f}\,dx = \left[\frac{x}{2} \cdot -\cos(x)\right]_0^{\pi} - \int_0^{\pi} \frac{-\cos(x)}{2}\,dx$$

$$= \frac{\pi}{2} \cdot 1 - \left[\frac{\sin(x)}{2}\right]_0^{\pi} = \frac{\pi}{2} - (0-0) = \frac{\pi}{2}$$

$$E(\cos(X)) = \int_{-\infty}^{+\infty} \cos(x) f(x)\,dx = \int_0^{\pi} \cos(x)\sin(x) \cdot \frac{1}{2}\,dx$$

$$= \frac{1}{2}\left[\frac{\sin^2(x)}{2}\right]_0^{\pi} = 0$$

c) $$E(X^2) - \frac{\pi^2}{4} = \frac{\pi^2}{4} - 2 \Leftrightarrow E(X^2) = \frac{\pi^2}{2} - 2$$

d) $$V(5X-4) = 5^2 V(X) = 25\left(\frac{\pi^2}{4} - 2\right) = \frac{25\pi^2}{4} - 50$$

2.14) $E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^1 x\frac{x}{2}\,dx + \int_1^2 x\frac{1}{2}\,dx + \int_2^3 x\left(\frac{3-x}{2}\right)dx$

$= \left[\frac{x^3}{6}\right]_0^1 + \left[\frac{x^2}{4}\right]_1^2 + \left[\frac{3}{4}x^2 - \frac{x^3}{6}\right]_2^3$

$= \frac{1}{6} + \frac{2^2}{4} - \frac{1}{4} + \frac{3.3^2}{4} - \frac{3^3}{6} - \frac{3.2^2}{4} + \frac{2^3}{6}$

$= \frac{3}{2}$

$E(X-1) = E(X) - 1 = \frac{1}{2}$

$V(X) = \frac{8}{3} - \left(\frac{3}{2}\right)^2 = \frac{8}{3} - \frac{9}{4} = \frac{5}{12}$

$E(X^2) = \int_0^1 x^2 \cdot \frac{x}{2}\,dx + \int_1^2 x^2 \cdot \frac{1}{2}\,dx + \int_2^3 x^2\left(\frac{3-x}{2}\right)dx$

$= \left[\frac{x^4}{8}\right]_0^1 + \left[\frac{x^3}{6}\right]_1^2 + \left[\frac{3}{6}x^3 - \frac{x^4}{8}\right]_2^3$

$= \frac{1}{8} + \frac{2^3}{6} - \frac{1}{6} + \frac{3.3^3}{6} - \frac{3^4}{8} - \frac{3.2^3}{6} + \frac{2^4}{8}$

$= \frac{8}{3}$

$E(X(X-1)) = E(X^2 - X) = E(X^2) - E(X) = \frac{8}{3} - \frac{3}{2} = \frac{7}{6}$

$E(e^X) = \int_{-\infty}^{+\infty} e^x f(x) = \int_0^1 \underset{f'}{e^x} \cdot \underset{g}{\frac{x}{2}}\,dx + \int_1^2 e^x \cdot \frac{1}{2}\,dx + \int_2^3 \underset{f'}{e^x}\underset{g}{\left(\frac{3-x}{2}\right)}dx$

$= \left[e^x \cdot \frac{x}{2}\right]_0^1 - \int_0^1 e^x \cdot \frac{1}{2}\,dx + \left[\frac{e^x}{2}\right]_1^2 + \left[e^x\left(\frac{3-x}{2}\right)\right]_2^3 - \int_2^3 e^x \cdot -\frac{1}{2}\,dx$

$= \cancel{\frac{e}{2}} - \left[\frac{e^x}{2}\right]_0^1 + \frac{e^2}{2} - \cancel{\frac{e}{2}} + e^2\left(\frac{3-2}{2}\right) - \left[-\frac{e^x}{2}\right]_2^3$

$= -\frac{e}{2} + \frac{1}{2} + \cancel{\frac{e^2}{2}} - \cancel{\frac{e^2}{2}} + \frac{e^3}{2} - \frac{e^2}{2} \approx 5,489$

$$F(\text{med}(x)) = \int_{-\infty}^{\text{med}(x)} f(x) = 0,5$$

$$F(1) = \int_0^1 \frac{x}{2}\,dx = \frac{1}{4} \neq 0,5$$

$$F(2) = \int_0^1 \frac{x}{2}\,dx + \int_1^2 \frac{1}{2}\,dx = \frac{1}{4} + 1 - \frac{1}{2} = \frac{3}{4} > 0,5$$

então $\text{med}(x) \in \,]1;2]$

$$F(\text{med}(x)) = \int_0^1 \frac{x}{2}\,dx + \int_1^{\text{med}(x)} \frac{1}{2}\,dx = \frac{1}{4} + \left[\frac{x}{2}\right]_1^{\text{med}(x)}$$

$$= \frac{1}{4} + \frac{\text{med}(x)}{2} - \frac{1}{2} = -\frac{1}{4} + \frac{\text{med}(x)}{2}$$

$$-\frac{1}{4} + \frac{\text{med}(x)}{2} = \frac{1}{2} \Leftrightarrow \text{med}(x) = \frac{3}{2}$$

$$CV = \frac{\sqrt{V(X)}}{E(X)} \times 100 = \frac{\sqrt{5/12}}{3/2} \times 100 = 43,03\%$$

2.16) $f(x) = \begin{cases} kx^3(1-x) & , 0 \leq x \leq 1 \\ 0 & , \text{c.c.} \end{cases}$

a) 1. $f(x) \geq 0 \quad \forall x \in \mathbb{R}$

para $x \in [0;1]$

$kx^3(1-x) \geq 0 \Rightarrow k \geq 0$

2. $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$$\int_0^1 kx^3(1-x)\,dx = 1 \Leftrightarrow k\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 = 1 \Leftrightarrow k\left(\frac{1}{4} - \frac{1}{5}\right) = 1$$

$$\Leftrightarrow \frac{k}{20} = 1 \Rightarrow k = 20$$

b) para $x < 0$ $F(x) = P(X = x) = 0$

para $0 \le x < 1$ $F(x) = P(X = x) = \int_0^x 20t^3(1-t)\,dt$

$= 20\left[\frac{t^4}{4} - \frac{t^5}{5}\right]_0^x = 20\left(\frac{x^4}{4} - \frac{x^5}{5}\right) = x^4(5-4x)$

para $x \geq 1$ $F(x) = P(X = x) = 1$

$$F(x) = \begin{cases} 0 & , x < 0 \\ x^4(5-4x) & , 0 \le x < 1 \\ 1 & , x \geq 1 \end{cases}$$

c) $V(5X-1) = 5^2 V(X) = 5^2\left(\frac{10}{21} - \frac{4}{9}\right) = \frac{50}{63}$

$E(X) = \int_0^1 x^4 20(1-x)\,dx = 20\left[\frac{x^5}{5} - \frac{x^6}{6}\right]_0^1 = 20\left(\frac{1}{5} - \frac{1}{6}\right) = \frac{2}{3}$

d) $L = \begin{cases} 1 - 0{,}8 = 0{,}2 & 1{,}2 - 0{,}8 = 0{,}4 \end{cases}$

$P(L = 0{,}4) = P(V = 1{,}2) = P(1/3 \le X \le 2/3) = \int_{1/3}^{2/3} f(x)\,dx$

$= F(2/3) - F(1/3)$

$= \left(\frac{2}{3}\right)^4\left(5 - 4 \cdot \frac{2}{3}\right) - \left(\frac{1}{3}\right)^4 \cdot \left(5 - \frac{4}{3}\right)$

$=$

Ex da sebenta

Função de uma v.a.

Considera-se X uma v.a. com função distribuição

$$F_X(x) = \begin{cases} 0 & , x \leq 0 \\ 5x^4 - 4x^3 & , 0 < x < 1 \\ 1 & , x \geq 1 \end{cases}$$

Identifica-se a distribuição das v.a.'s $Y = 2X-1$ e $W = X^2$ (através da função distribuição)

$$F_Y(Y) = P(Y \leq y) = P(2X-1 \leq y) = P\left(X \leq \frac{y+1}{2}\right) = F_X\left(\frac{y+1}{2}\right)$$

Logo

$$F_Y(y) = \begin{cases} 0 & , \frac{y+1}{2} \leq 0 \Leftrightarrow y \leq -1 \\ 5\left(\frac{y+1}{2}\right)^4 - 4\left(\frac{y+1}{2}\right)^3 & , 0 < \frac{y+1}{2} < 1 \quad (-1 < y < 1) \\ 1 & , \frac{y+1}{2} \geq 1 \Leftrightarrow y \geq 1 \end{cases}$$

$$F_W(w) = P(W \leq w) = P(X^2 \leq w) = P(X \leq \pm w) = P(-\sqrt{w} \leq X \leq \sqrt{w})$$
$$= P(X \leq \sqrt{w}) - P(X \leq -\sqrt{w}) = F(\sqrt{w}) - F(-\sqrt{w}) = F_X(\sqrt{w})$$

$$F_W(w) = \begin{cases} 0 & , w \leq 0 \\ 5(\sqrt{w})^4 - 4(\sqrt{w})^3 & , 0 < w < 1 \\ 1 & , w \geq 1 \end{cases}$$

Exemplo 3.7 da sebenta

A funç. de prob. marginal de X e Y

$$X = \begin{cases} 0 & 1 & 2 \\ 3/8 & 3/8 & 1/4 \end{cases}$$

$$Y = \begin{cases} 0 & 1 & 2 \\ 3/8 & 1/4 & 3/8 \end{cases}$$

$$X|Y=2 \begin{cases} 0 & 1 & 2 \\ 0 & \underbrace{1/3}_{\frac{P(X=1)}{P(Y=2)}} & \frac{2}{3} \, \frac{P(X=2)}{P(Y=2)} \end{cases}$$

$$E(X|Y=2) = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$

$$V(X) = E\left[(X - E(X))^2\right] = E(X^2) - E^2(X)$$

$$Cov(X,Y) = E\left[(X - E(X))(Y - E(Y))\right]$$

avalia a variabilidade conjunta das duas variáveis

pos - variam no mesmo sentido
neg - varia em sentidos contrários

① $Cov(X,Y) = E(XY) - E(X)E(Y)$

$$\begin{aligned} Cov(X,Y) &= E\left[(X - E(X))(Y - E(Y))\right] \\ &= E\left[XY - XE(Y) - YE(X) + E(X)E(Y)\right] \\ &= E(XY) - E(XE(Y)) - E(YE(X)) + E(E(X)E(Y)) \\ &= E(XY) - E(X)E(Y) - \cancel{E(Y)E(X)} + \cancel{E(X)E(Y)} \end{aligned}$$

$\therefore$

② se X e Y forem v.a. independentes então

$$E(XY) = E(X)E(Y) \Rightarrow Cov(X,Y) = 0$$

Dem: se X e Y forem independentes

$$P(X = x_i ; Y = y_i) = P(X = x_i) P(Y = y_i)$$

logo

$$E(XY) = \sum_i \sum_j x_i y_j P(X = x_i) P(Y = y_j)$$

$$= \sum_i x_i P(X = x_i) \cdot \sum_j y_j P(Y = y_j)$$

$$= E(X) \cdot E(Y)$$

③ $Cov(a + bX, c + dY) = bd\,Cov(X,Y)$

Seja X e Y com variância $V(X)$ e $V(Y)$

então

$$V(X \pm Y) = V(X) + V(Y) \pm 2\,Cov(X,Y)$$

Dem

$$V(X+Y) = E[(X+Y)^2] - E^2(X+Y)$$

Coeficiente de correlação de (X,Y) por $\rho(X,Y) = \dfrac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}$

3.1) a) $15c = 1 \Rightarrow c = \frac{1}{15}$

$$Y = \begin{cases} 0 & 2 & a \\ 1/5 & 1/3 & 7/15 \end{cases} \qquad \frac{38}{15} = \frac{2}{3} + \frac{7}{15}a \Rightarrow a = 4$$

$$E(Y(Y-1)) = E(Y^2 - Y) = E(Y^2) - E(Y) = \frac{44}{5} - \frac{38}{15} = \frac{94}{15}$$

$$E(Y^2) = \frac{4}{3} + 4^2 \cdot \frac{7}{15} = \frac{44}{5}$$

$$P(X + Y \geq 3) = P(X=0 ; Y=4) + P(X=1; Y=2) + P(X=1; Y=4) = \frac{2}{3}$$

b) $P(X=1 \mid Y=2) = \frac{P(X=1; Y=2)}{P(Y=2)} = \frac{1/5}{13} = 3/5$

c) $Cov(X,Y) = E(XY) - E(X)E(Y) = -\frac{4}{75}$

$E(X) = \frac{3}{5}$

$E(Y) = \frac{38}{15}$

$E(XY) = 0 \cdot 0 \cdot \frac{1}{15} + 1 \cdot 0 \cdot \frac{2}{15} + 1 \cdot 2 \cdot \frac{3}{15} + \dots = \frac{22}{15}$

3.2) a)

X\Y	0	1	2	
0	0	0,1	0,15	0,25
1	0,1	0,2	0,05	0,35
2	0,1	0,1	0,1	0,3
3	0	0,1	0	0,1
	0,2	0,5	0,3	1

$0,25 + 0,35 + p + 0,2 + p = 1$

$\Rightarrow p = 0,1$

$P(X < Y) = 0,1 + 0,15 + 0,05 = 0,3$

b) $(X \mid Y=1) = \begin{cases} 0 & 1 & 2 & 3 \\ \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & \frac{1}{5} \end{cases}$

$E(X \mid Y=1) = \frac{2}{5} + \frac{2}{5} + \frac{3}{5} = \frac{7}{5}$

c) $P(X=1 \mid Y=1) + P(X=2 \mid Y=1) = \frac{3}{5}$

d) $E(X) = 1 \times 0,35 + 2 \times 0,3 + 3 \times 0,1 = \frac{5}{4}$

e) $X+Y = \begin{cases} 1 & 2 & 3 & 4 \\ 0,2 & 0,45 & 0,15 & 0,2 \end{cases}$

g) $V(X) = 0,8875$

$Cov(X,Y) = \underbrace{E(XY)}_{\frac{6}{5}} - \underbrace{E(X)}_{\frac{5}{4}}\underbrace{E(Y)}_{\frac{11}{10}} = -0,175$

$V(X-2Y) = \underbrace{V(X)}_{0,8875} + 2^2 \underbrace{V(Y)}_{0,49} + 2\cdot -2 \underbrace{Cov(X,Y)}_{-0,175} = 3,5475$

$\rho(X,Y) = \dfrac{Cov(X,Y)}{\sqrt{V(X)V(Y)}} = \dfrac{-0,175}{\sqrt{0,434875}} = -0,2654$

f) $E(X+Y) = 0,2 + 2\cdot 0,45 + \cdots = 2,35$

3.3) a)

X\Y	0	1	
0	0,02	0,14	0,16
1	0,24	0,24	0,48
2	0,24	0,12	0,36
	0,5	0,5	1

b) Não $P(X=2; Y=1) = 0,12 \neq 0,18 = P(X=2).P(Y=1)$

c) $V(Y-2X) = V(Y) + 2^2 V(X) - Cov(X,Y) = 2,57$

$V(Y) = 0,5 - 0,25 = 0,25$

$V(X) = 1,92 - 1,44 = 0,48$

$Cov(X,Y) = 0,2016 - 1,2 \times 0,5 = -0,3984$

d)

3.4) a)

X\Y	0	1	
0	0,72	0,06	0,78
1	0,08	0,14	0,22
	0,8	0,2	1

$P(X=1 \mid Y=1) = \dfrac{P(X=1; Y=1)}{P(Y=1)}$

$\Rightarrow P(X=1; Y=1) = 0,14$

$P(X=1 \mid Y=0) \Rightarrow P(X=1; Y=0) = 0,08$

b) Não são independentes

c) $P(X<Y) = P(X=0; Y=1) = 0,06$

d) $P(X+Y<2) = 0,72 + 0,06 + 0,08 = 0,86$

3.5)

$X \backslash Y$	0	1	2	
0	0,35	0,14	0,21	0,7
1	0,15	0,06	0,09	0,3
	0,5	0,2	0,3	1

3.6) a) $\rho(X,Y) = \dfrac{Cov(X,Y)}{\sqrt{V(X)V(Y)}} = \dfrac{\sigma^2}{\sqrt{2\sigma^4}} = \dfrac{\cancel{\sigma^2}}{\cancel{\sigma^2}\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$V(W) = V(X-Y) = \underbrace{\cancel{V(X)}}_{\cancel{\sigma^2}} + \underbrace{V(Y)}_{2\sigma^2} - 2Cov(X,Y) = \cancel{\sigma^2} \Rightarrow Cov(X,Y) = \sigma^2$

b) $V(T) = V(2X+Y) = 4V(X) + V(Y) - 2 \cdot 2 Cov(X,Y) = 4\sigma^2 + 2\sigma^2 + 4\sigma^2 = 10\sigma^2$

c) $Cov(W,T) = Cov(X-Y, 2X+Y) = 2Cov(X,X) + Cov(X,Y) - 2Cov(Y,X) - Cov(Y,Y)$

$= 2V(X) + Cov(X,Y) - 2Cov(X,Y) - V(Y) = 2\cancel{\sigma^2} + \sigma^2 - 2\sigma^2 - \cancel{2\sigma^2} = -\sigma^2$

3.7) $V(X) = V(Y) = \sigma^2$

$V(W) = V(X-Y) = V(X) + V(Y) - 2Cov(X,Y)$

$= 2\sigma^2 - 2\sigma^2\rho = 2\sigma^2(1-\rho)$

$\underbrace{\rho(X,Y)}_{\rho} = \dfrac{Cov(X,Y)}{\sqrt{V(X)V(Y)}} \Leftrightarrow Cov(X,Y) = \rho\sigma^2$

$Cov(U,W) = Cov(X+Y, X-Y)$

$= Cov(X,X) - Cov(X,Y) + Cov(Y,X) - Cov(Y,Y)$

$= V(X) - V(Y) = 0$

4.1) X - nº de latas com o prazo ultrapassado em 6

$X \sim H(\underset{N}{50}, \underset{M}{7}, \underset{n}{6})$

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X=0) = 1 - \frac{\binom{7}{0}\binom{43}{6}}{\binom{50}{6}} = 0,6164$

4.2) X - praias inspeccionadas em bom estado em 10

$X \sim H(\underset{N}{100}, \underset{M}{85}, \underset{n}{10})$

a) $P(X=7) = \frac{\binom{85}{7}\binom{15}{3}}{\binom{100}{10}} = 0,1297$

b) $P(X \geq 8) = 1 - P(X \leq 8)$

c) Y - praias inspeccionadas em mau estado em 5

$Y \sim H(\underset{N}{100}, \underset{M}{15}, \underset{n}{5})$

$P(Y=0) = \frac{\binom{15}{0}\binom{85}{5}}{\binom{100}{5}} = 0,4357$

d) $E(X) = \cancel{10} \cdot \frac{85}{10\cancel{0}} = 8,5$ praias

4.3) X - nº de selos que valem 100 € em 5

$X \sim H(\underset{N}{20}, \underset{M}{6}, \underset{n}{5})$

a) $P(X=0) = \frac{\binom{6}{0}\binom{14}{5}}{\binom{20}{5}} = 0,1291$

b) $P(X=1) = \frac{\binom{6}{1}\binom{14}{4}}{\binom{20}{5}} = 0,3874$

4.4) X - nº de pessoas a pedir refeição vegetariana em 10

$X \sim Bin(\underset{n}{10}, \underset{p}{0,2})$

a) $P(X=0) = \binom{10}{0} 0,2^0 \, 0,8^{10} = 0,1074$

b) $P(X=10) = \binom{10}{10} 0,2^{10} \, 0,8^0 = 1,024 \times 10^{-7}$

c) $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X=0) = 1 - 0,1074 = 0,8926$

4.5) X - nº de respostas correctas em 5

$X \sim Bin(5, \frac{1}{4})$ (n, p)

$P(X \geq 3) = 1 - P(X < 3) = 1 - P(X=0) - P(X=1) - P(X=2)$

$= 1 - \binom{5}{0} 0{,}25^0 \cdot 0{,}75^5 - \binom{5}{1} 0{,}25^1\, 0{,}75^4 - \binom{5}{2} 0{,}25^2\, 0{,}75^3$

$= 0{,}1035$

$E(X) = 1{,}25$ respostas

$\sigma(X) = \sqrt{1{,}25 \cdot \frac{3}{4}} = 0{,}9682$ respostas

4.6) X - nº de copos com defeito em 50.

$X \sim Bin(50, 0.05)$ (n, p)

a) $P(X=0) = \binom{50}{0} 0.05^0\, 0.95^{50} = 0{,}0769$

b) $P(X=1) = \binom{50}{1} 0.05^1\, 0.95^{49} = 0{,}2025$

c) $P(X \leq 1) = P(X=0) + P(X=1) = 0{,}2794$

d) $E(X) = 2{,}5$ copos

$\sigma(X) = \sqrt{2{,}5 \cdot 0{,}95} = 0{,}4873$ copos

4.7) X - nº de caras em 10 lançamentos

Y - nº de caras em 15 lançamentos

$X \sim Bin(10, 0.5)$

$Y \sim Bin(15, 0.5)$

$Z = X + Y \sim Bin(25, 0.5)$

$P(Z = 12) = \binom{25}{12} 0.5^{12}\, 0.5^{13} = 0{,}1550$

4.8) a) $\begin{cases} p = 2q \\ p + q = 1 \end{cases} \Rightarrow \begin{cases} p = 2/3 \\ q = 1/3 \end{cases}$

$X \sim Bin(4, 2/3)$

$P(X=k) = \binom{4}{k}\left(\frac{2}{3}\right)^k\left(\frac{1}{3}\right)^{n-k}$, $k = 0,1,2,3,4$

b) $Y \sim Geo(1/3)$

i) $E(Y) = 3$

$CV(Y) = \frac{\sqrt{V(X)}}{E(X)} = 81,65\%$

moda = 1

ii) $P(Y=4) = \frac{1}{3}\cdot\left(\frac{2}{3}\right)^3 = 0,0988$

iii) $P(Y \geq 2) = 1 - P(Y<2) = 1 - P(Y=1)$

$= 1 - \frac{1}{3}\cdot\left(\frac{2}{3}\right)^0 = \frac{2}{3}$

4.9) X - nº de presos ~~que~~ homicidas

$X \sim H(\underset{N}{1500}, \underset{M}{60}, \underset{n}{8})$

Como $\frac{n}{N} = 0,005 < 0,1$ então podemos fazer uma aproximação utilizando a distribuição Binomial com $n = 8$ e $p = 0,04$

$P(X=2) \simeq \binom{8}{2} 0.04^2 \cdot 0.96^6 = 0,0351$

4.10) X - nº de clientes que compram um produto da empresa em 25

$X \sim H(\underset{N}{1000}, \underset{M}{300}, \underset{n}{25})$

Como $\frac{n}{M} = 0,025 < 0,1$ então podemos fazer uma aproximação utilizando a distribuição binomial com $n = 25$ e $p = \frac{3}{10}$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$\simeq \binom{25}{0}\left(\frac{3}{10}\right)^0\left(\frac{7}{10}\right)^{25} + \binom{25}{1}\left(\frac{3}{10}\right)^1\left(\frac{7}{10}\right)^{24} + \binom{25}{1}\left(\frac{3}{10}\right)^2\left(\frac{7}{10}\right)^{23}$